MATH1034OL1 Pre-Calculus Mathematics Notes from Section 2.5 (Friday)

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Contents

1 Inverse Functions

1.1 Finding Inverses

Given a function $f(x)$, we denote its inverse as $f^{-1}(x)$. To find the inverse of $f(x)$:

1. Replace $f(x)$ with y.
2. Switch y and x.
3. Solve for y.
4. Replace y with $f^{-1}(x)$

Here's an example:

Problem: Let $f(x) = x^2 + 3$. Find $f^{-1}(x)$.

Replace $f(x)$ with y: $f(x) = x^2 + 3 \implies y = x^2 + 3$

Switch x and y: $y = x^2 + 3 \implies x = y^2 + 3$

Solve for y: $x = y^2 + 3 \implies y^2 = x - 3 \implies y = \sqrt{x - 3}$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt{x - 3}$

Now we know $f^{-1}(x) = \sqrt{x - 3}$.

1.2 Properties of Inverses

A cool property of inverse functions is that $f(x)$ and $f^{-1}(x)$ reflect over the line $y = x$:

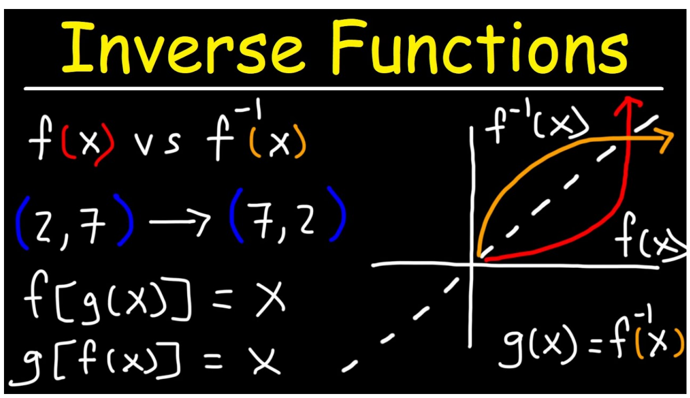

Figure 1: Credit: https://youtu.be/TN4ybFiuV3k

This makes sense because we are switching x and y.

Another property of f^{-1} is that its domain and range are the range and domain respectfully of f. More formally:

domain(f) = range(f^{-1})

range(f) = domain(f^{-1})

2 Factoring Higher-Degree Polynomials

Let's suppose we have the equation $8x^3 + x + 9$=0. The left side isn't easily factorable and can't be plugged into the quadratic formula. Instead, we can find the possible rational zeros of the expression.

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$ where a are integers. Here,

$$\text{possible rational zeros} = \frac{\text{factors of } a_0 \text{ (last term)}}{\text{factors of } a_n \text{(first term)}}$$

Hence, the possible rational zeroes of $8x^3 + x + 9$ are $\pm\frac{1,3,9}{1,2,4,8} = \pm\frac{1}{8}, \pm\frac{1}{4}, \pm\frac{3}{8}, \pm\frac{1}{2}, \pm\frac{3}{4}, \pm 1, \pm\frac{9}{8}, \pm\frac{3}{2}, \pm\frac{9}{4}, \pm\frac{9}{2}, \pm 3, \pm 9$.

Personally, I like to start with integers. We can manually plug possible rational zeros in as x. However, there's a trick to quickly evaluate polynomials at a certain value x. This isn't easily explained in writing, so here's a helpful video from the Organic Chemistry tutor.

Just remember that if $f(a) = 0$ than $(x - a)$ is a factor of f.

3 Midterm Concepts

If you study these, you will do well on the midterm exam:

1. Converting between degrees and radians
2. Determining reference angles
3. All Students Take Calculus
4. Unit Circle $(x, y) = (\cos, \sin)$; $\sec = \frac{1}{x}$, $\csc \frac{1}{y}$; $\tan = \frac{\sin}{\cos} = \frac{y}{x}$; $\cot = \frac{\cos}{\sin} = \frac{x}{y}$
5. 30-60-90 triangle properties
6. 45-45-90 triangle properties
7. Trigonometric functions in terms of the sides of an angle:

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cot\theta = \frac{\text{adj}}{\text{opp}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}}$$

8. Composing functions and function operations
9. Inverse functions
10. Solving high degree polynomials and the rational zero theorem
11. Equations of circles