

MATH1034OL1 Pre-Calculus Mathematics Notes from Sections 4.8, 3.3 (Monday)

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1 Inverse Trigonometric Functions

The three inverse trigonometric functions are \arccos , \arcsin , and \arctan . These functions have the property

$$\begin{aligned}\arccos(x) &= A \\ \cos(A) &= x\end{aligned}$$

In other words, \cos and \arccos are inverses of each other. They're helpful for determining an angle given its \sin , \cos , or \tan value.

When we're finding $\arcsin(x)$, we restrict the range of \arcsin such that it remains a function (that is, there are no two outputs $\arcsin(x)$ for one input x). We do this for all inverse

trigonometric functions. Here are the restrictions:

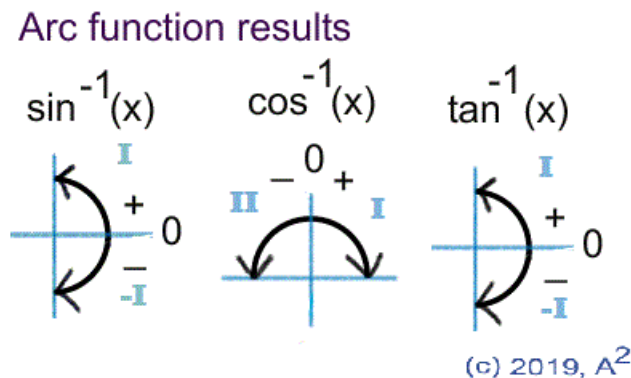


Figure 1: Credit: <https://www.mathnstuff.com/math/spoken/here/2class/330/arc.htm>

Problem: Find the Exact value of $\sec\{\arctan[\sin(\arccos(\frac{-1}{2}))]\}$.

First, recall that $\arccos(\frac{-1}{2})$ will be the angle whose cosine value is $\frac{-1}{2}$. We know this angle is $\frac{2\pi}{3}$ using the rules above.

Next, we determine that $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$. This is where things get a little more complicated. $\arctan(\frac{\sqrt{3}}{2})$ isn't an angle with reference angle $30^\circ, 45^\circ, 60^\circ, 90^\circ$. This is fine because, when we zoom out, we're asked to find $\sec(\arctan(\frac{\sqrt{3}}{2}))$. We know that the triangle formed by the angle $\arctan(\frac{\sqrt{3}}{2})$ has opp = $\sqrt{3}$ and adj = 2. We use the pythagorean theorem to find the hypotenuse:

$$\begin{aligned} (\sqrt{3})^2 + 2^2 &= c^2 \\ \implies c &= \sqrt{3 + 4} = \sqrt{7} \end{aligned}$$

Hence, hyp = $\sqrt{7}$. Even though the angle $\arctan(\frac{\sqrt{3}}{2})$ doesn't form a special triangle, we can still find its sec value using the sides of the triangle it forms:

$$\sec\left(\arctan\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{7}}{2}$$

Knowing this, $\sec\{\arctan[\sin(\arccos(\frac{-1}{2}))]\} = \frac{\sqrt{7}}{2}$.

2 Polynomial Long Division

Polynomial division isn't easily explained in notes, but it's very important!. Here is my recommended resource for polynomial division:

<https://www.mathsisfun.com/algebra/polynomials-division-long.html>

3 Remainder Theroem

Remainder Theorem Formula



When $p(x)$ is divided by $(x - a)$

$$\text{Remainder} = p(a)$$

OR

When $p(x)$ is divided by $(ax + b)$

$$\text{Remainder} = p\left(-\frac{b}{a}\right)$$

Figure 2: Credit: <https://www.cuemath.com/algebra/remainder-theorem/>

4 Descartes's Rule of Signs

Descartes' Rule of Signs



Number of positive real roots of $f(x)$
= Number of sign changes of $f(x)$
(OR)
< Number of sign changes of $f(x)$
by even number

Number of negative real roots of $f(x)$
= Number of sign changes of $f(-x)$
(OR)
< Number of sign changes of $f(-x)$
by even number

Figure 3: Credit: <https://www.cuemath.com/algebra/descartes-rule-of-signs/>

Descartes' Rule of Signs

$$+x^5 - 2x^4 - 3x^3 + 4x^2 - x - 1$$

3 changes → 3 or 1 positive real solutions

$$(-x)^5 - 2(-x)^4 - 3(-x)^3 + 4(-x)^2 - (-x) - 1$$
$$-x^5 - 2x^4 + 3x^3 + 4x^2 + x - 1$$

2 changes → 2 or 0 negative real solutions




Figure 4: Credit: <https://andymath.com/descartes-rule-of-signs/>