MATH1034OL1 Pre-Calculus Mathematics Notes from Sections 4.8, 3.3 (Monday)

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Contents

1 Inverse Trigonometric Functions

The three inverse trigonometric functions are arccos, arcsin, and arctan. These functions have the property

$$\arccos(x) = A$$
$$\cos(A) = x$$

In other words, cos and arccos are inverses of each other. They're helpful for determining an angle given its $\sin, \cos$, or $\tan$ value.

When we're finding $\arcsin(x)$, we restrict the range of arcsin such that it remains a function (that is, there are no two outputs $\arcsin(x)$ for one input x). We do this for all inverse trigonometric functions. Here are the restrictions:

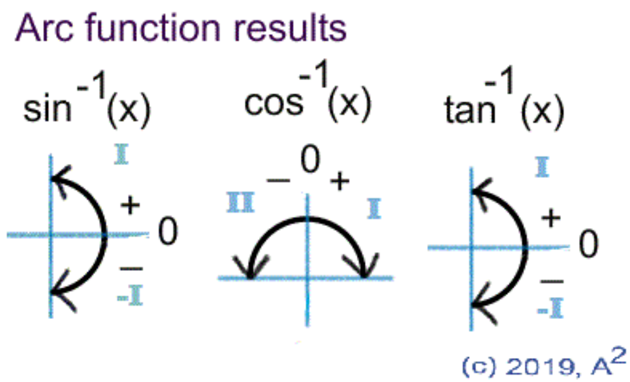

Figure 1: Credit: https://www.mathnstuff.com/math/spoken/here/2class/330/arc.htm

Problem: Find the Exact value of $\sec\{\arctan[\sin(\arccos(\frac{-1}{2}))]\}$.

First, recall that $\arccos(\frac{-1}{2})$ will be the angle whose cosine value is $\frac{-1}{2}$. We know this angle is $\frac{2\pi}{3}$ using the rules above.

Next, we determine that $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$. This is where things get a little more complicated. $\arctan(\frac{\sqrt{3}}{2})$ isn't an angle with reference angle $30°, 45°, 60°, 90°$. This is fine because, when we zoom out, we're asked to find $\sec(\arctan(\frac{\sqrt{3}}{2}))$. We know that the triangle formed by the angle $\arctan(\frac{\sqrt{3}}{2})$ has opp $= \sqrt{3}$ and adj $= 2$. We use the pythagorean theorem to find the hypotenuse:

$$(\sqrt{3})^2 + 2^2 = c^2$$
$$\implies c = \sqrt{3+4} = \sqrt{7}$$

Hence, hyp $= \sqrt{7}$. Even though the angle $\arctan(\frac{\sqrt{3}}{2})$ doesn't form a special triangle, we can still find its sec value using the sides of the triangle it forms:

$$\sec\left(\arctan\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{\text{hyp}}{\text{adj}} = \frac{\sqrt{7}}{2}$$

Knowing this, $\sec\{\arctan[\sin(\arccos(\frac{-1}{2}))]\} = \frac{\sqrt{7}}{2}$.

2 Polynomial Long Division

Polynomial division isn't easily explained in notes, but it's very important!. Here is my recommended resource for polynomial division:

https://www.mathsisfun.com/algebra/polynomials-division-long.html

3 Remainder Theroem

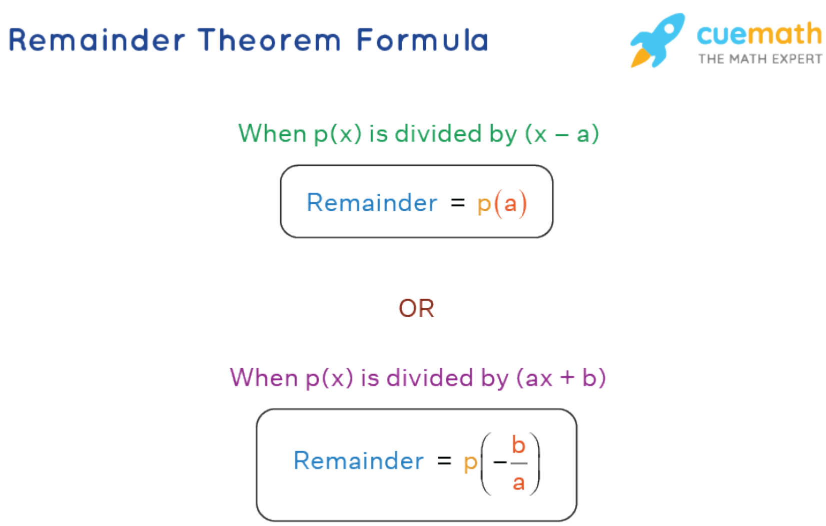

Figure 2: Credit: https://www.cuemath.com/algebra/remainder-theorem/

4 Descarte's Rule of Signs

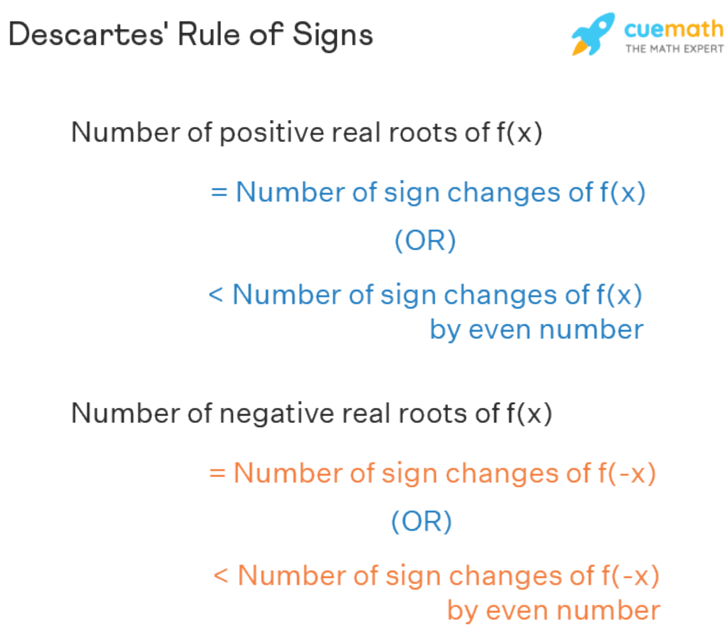

Figure 3: Credit: https://www.cuemath.com/algebra/descartes-rule-of-signs/

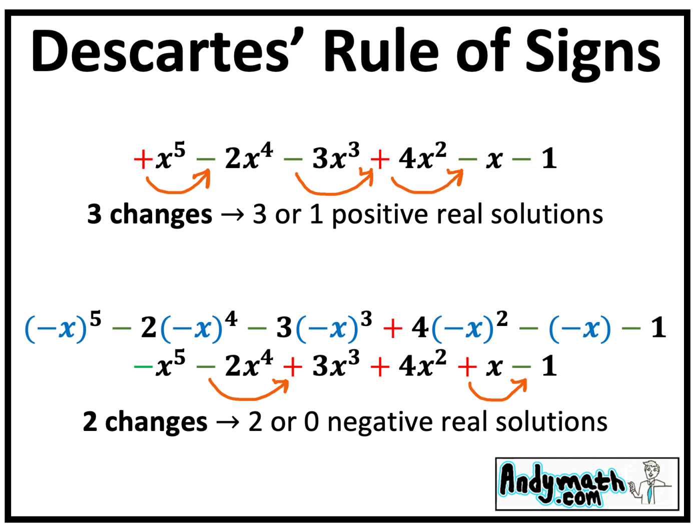

Figure 4: Credit: https://andymath.com/descartes-rule-of-signs/