

# MATH1034OL1 Pre-Calculus Mathematics Notes from Sections 5.2, 5.3 (Friday)

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August 2, 2024

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## 1 Compound Interest and the Natural Base

For an initial amount  $P$ , interest rate  $r$ , compounding rate per time period  $t$ , and time periods elapsed  $t$ , we define the final amount  $A$  as

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

If the compounding is continuous, meaning it's calculate infinitely many times in a given compounding period, the base is  $e \approx 2.718$ . Let's derive  $e$ :

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{1 \cdot n}$$

Hence, to calculate the new amount given an initial amount  $P$  after  $t$  time periods with interest rate  $r$ , we use  $A = Pe^{rt}$

For  $P = 1000$ ,  $t = 3$ , and  $r = 13\%$ :  $A = 1000e^{0.13 \cdot 3} \approx 1,476.98$

## 2 Logarithms

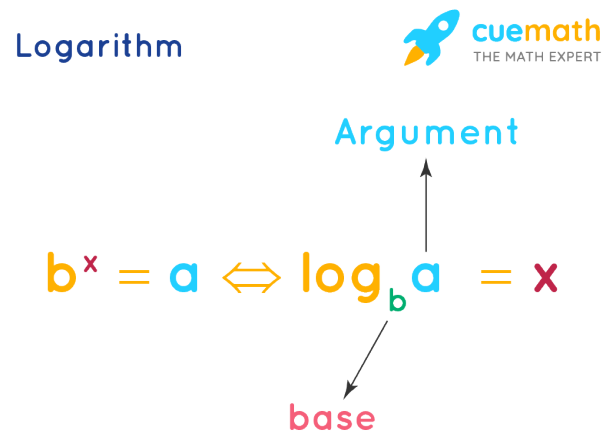


Figure 1: Credit: <https://www.cuemath.com/log-formulas/>

In plain language: the value of a logarithm is the power the base must be raised to in order to get the argument.

Logarithms are also the inverses of exponential functions. This means that for  $f(x) = b^x$ ,  $f^{-1}(x) = \log_b x$ . Given this property, we know that an exponential function and its corresponding inverse logarithmic function will reflect over the line  $y = x$ :

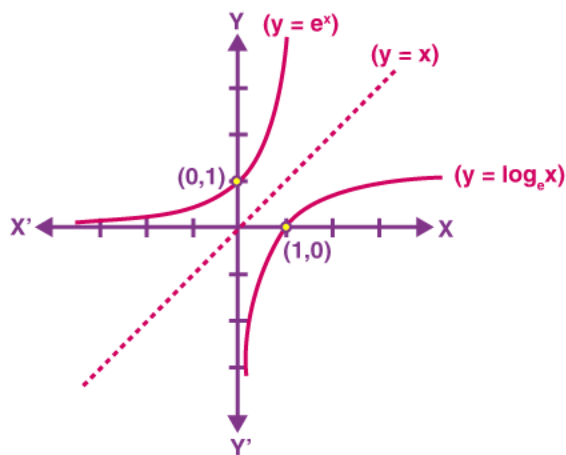


Figure 2: Credit: <https://byjus.com/maths/exponential-and-logarithmic-functions/>

Here are the rules to find the inverse of a logarithmic or exponential function:

$\pi$

### To find the inverse of a logarithmic function, follow these steps:

**Example:** Find the inverse of  $f(x) = \log_2(x+3)$ .

Step 1: write as  $y = \log_2(x+3)$ .

Step 2: swap  $x$  and  $y$ :  $x = \log_2(y+3)$ .

Step 3: make  $y$  the subject by changing to an exponential form first :

$$2^x = y + 3$$

Step 4: then subtract 3 from both sides:

$$2^x - 3 = y$$

Step 5: replace  $y$  with  $f^{-1}(x)$ .

Step 6: write down the final answer

$$f^{-1}(x) = 2^x - 3$$

Figure 3: Credit: <https://slideplayer.com/slide/12938368/>

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## To find the inverse of an exponential function, follow these steps:

**Example:** Find the inverse of  $f(x) = 3^{x-1} + 5$ .

Step 1: write as  $y = 3^{x-1} + 5$

Step 2: swap  $x$  and  $y$ :  $x = 3^{y-1} + 5$

Step 3: make  $y$  the subject by subtracting 5 from both sides first :

$$x - 5 = 3^{y-1}$$

Step 4: then take log base 3 of both sides:

$$\log_3(x - 5) = \log_3 3^{y-1}$$

$$\log_3(x - 5) = y - 1$$

Step 5: add 1 to both sides

$$\log_3(x - 5) + 1 = y$$

Step 5: replace  $y$  with  $f^{-1}(x)$ .

Step 6: write down the final answer

$$f^{-1}(x) = \log_3(x - 5) + 1$$

Figure 4: Credit: <https://slideplayer.com/slide/12938368/>

We didn't discuss logarithmic graphs in class, but here are some important properties to understand:

$|a| > 1 \rightarrow$  vertical stretch by a factor of  $a$        $h \rightarrow$  horizontal translation  
 $0 < |a| < 1 \rightarrow$  vertical compression by a factor of  $a$        $k \rightarrow$  vertical translation  
 $a < 0 \rightarrow$  reflection over the x-axis

$$y = a \cdot \log_b(x - h) + k$$

**Transformations  
of  
Logarithmic  
Functions**

Domain	Range	Vertical Asymptote
$[h, \infty)$ or $x \geq h$	$(-\infty, \infty)$	$x = h$

Figure 5: Credit: <https://goodsifyet.shop/product-details/7113278.html>

There exists an asymptote at  $x = h$  because logarithmic functions may not have an argument less than 0. Similarly, they also may not have a negative base.

There are some rules for logarithms, too:

## Rule of Logarithms



Rule Name	Property
Log of 1	$\log_b 1 = 0$
Log of the same number as base	$\log_b b = 1$
Product Rule	$\log_b(mn) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$
Change of Base Rule	$\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \cdot \log_c a = \log_c b$
Equality Rule	$\log_b a = \log_b c \Rightarrow a = c$
Number Raised to Log	$b^{\log_b x} = x$
Other Rules	$\log_b a^m = \frac{m}{n} \log_b a$ $-\log_b a = \log_b \frac{1}{a}$ (OR) $= \log_{\frac{1}{b}} a$

Figure 6: Credit: <https://www.cuemath.com/algebra/log-rules/>

## 3 Fraction Exponents

$$x^{\frac{a}{b}} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$$