MATH1034OL1 Pre-Calculus Mathematics Notes from Sections 5.2, 5.3 (Friday)

Elijah Renner

August 2, 2024

Contents

1 Compound Interest and the Natural Base

For an initial amount P, interest rate r, compounding rate per time period t, and time periods elapsed t, we define the final amount A as

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

If the compounding is continuous, meaning it's calculate infinitely many times in a given compounding period, the base is $e \approx 2.718$. Let's derive e:

$$e = \lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^{1 \cdot n}$$

Hence, to calculate the new amount given an initial amount P after t time periods with interest rate r, we use $A = Pe^{rt}$

For $P = 1000$, $t = 3$, and $r = 13\%$: $A = 1000e^{0.13 \cdot 3} \approx 1,476.98$

2 Logarithms

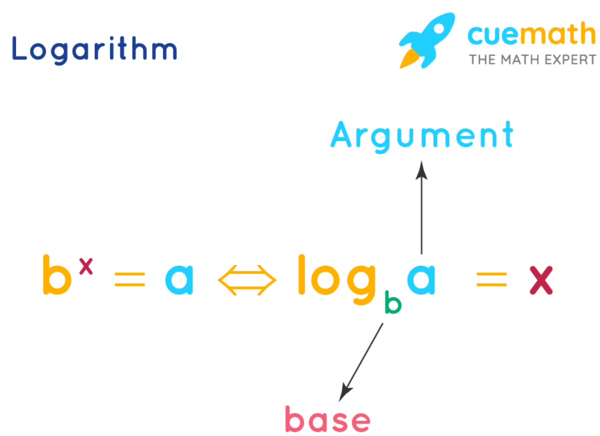

Figure 1: Credit: https://www.cuemath.com/log-formulas/

In plain language: the value of a logarithm is the power the base must be raised to in order to get the argument.

Logarithms are also the inverses of exponential functions. This means that for $f(x) = b^x$, $f^{-1}(x) = \log_b x$. Given this property, we know that a an exponential function and its corresponding inverse logarithmic function will reflect over the line $y = x$:

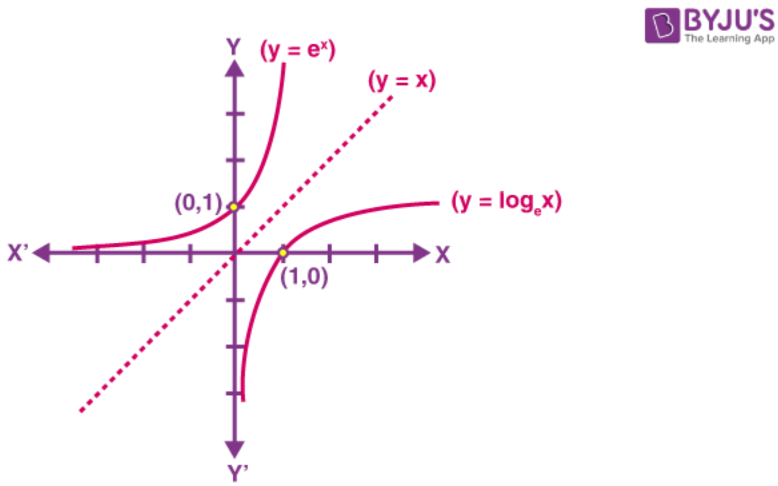

Figure 2: Credit: https://byjus.com/maths/exponential-and-logarithmic-functions/

Here are the rules to find the inverse of a logarithmic or exponential function:

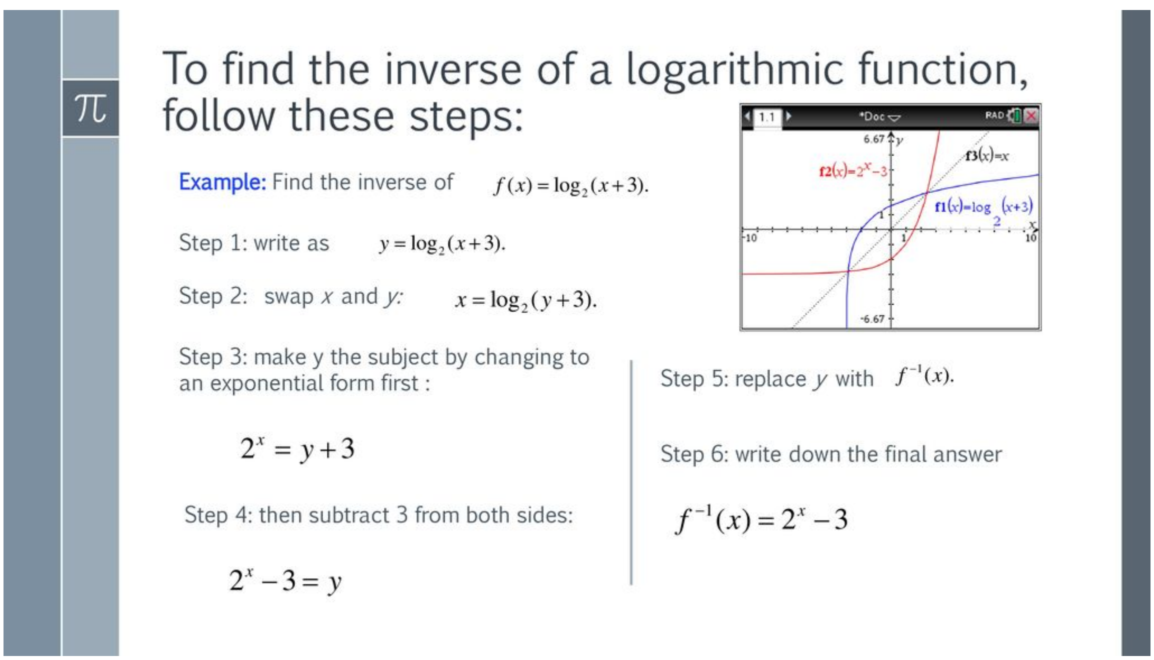

Figure 3: Credit: https://slideplayer.com/slide/12938368/

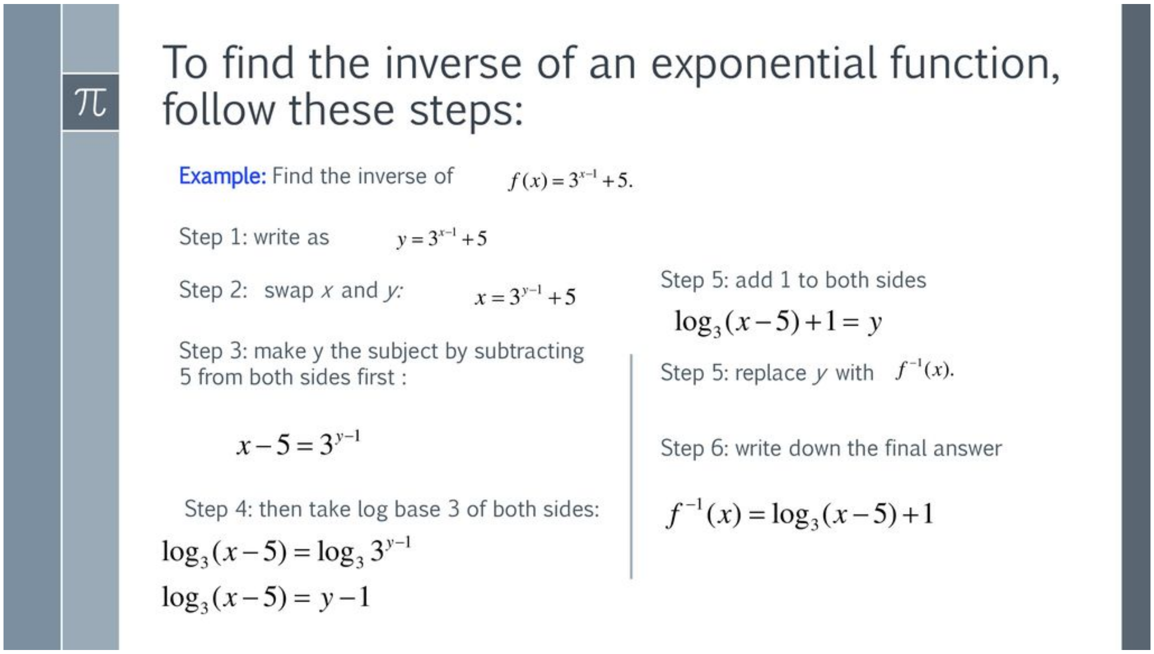

Figure 4: Credit: https://slideplayer.com/slide/12938368/

We didn't discuss logarithmic graphs in class, but here are some important properties to understand:

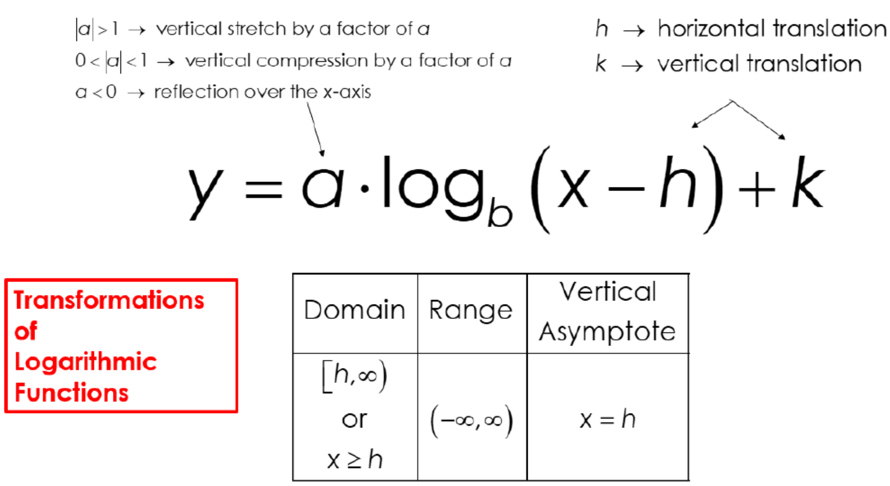

Figure 5: Credit: https://goodsifyet.shop/product_details/7113278.html

There exists an asymptote at $x = h$ because logarithmic functions may not have an argument less than 0. Similarly, they also may not have a negative base.

There are some rules for logarithms, too:

Rule of Logarithms

Rule Name	Property
Log of 1	$\log_b 1 = 0$
Log of the same number as base	$\log_b b = 1$
Product Rule	$\log_b(mn) = \log_b m + \log_b n$
Quotient Rule	$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
Power Rule	$\log_b m^n = n \log_b m$
Change of Base Rule	$\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \,.\, \log_c a = \log_c b$
Equality Rule	$\log_b a = \log_b c \Rightarrow a = c$
Number Raised to Log	$b^{\log_b x} = x$
Other Rules	$\log_{b^n} a^m = \frac{m}{n} \log_b a$ $-\log_b a = \log_b \frac{1}{a}$ (OR) $= \log_{\frac{1}{b}} a$

Figure 6: Credit: https://www.cuemath.com/algebra/log-rules/

3 Fraction Exponents

$$x^{\frac{a}{b}} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$$